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饱和土中管桩的水平动阻抗研究

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摘 要: 为了考察桩、土主要参数对饱和土中管桩水平振动的影响, 将土体分为桩周饱和土和桩芯饱和土两部分, 利用多孔介质理论的饱和土控制方程建立了饱和土-管桩的耦合振动模型。在考虑桩周饱和土和桩芯饱和土边界条件的情况下, 运用势函数解耦的方法对桩周饱和土和桩芯饱和土的水平振动进行了求解。在考虑桩周饱和土和桩芯饱和土对管桩作用的情况下对饱和土中管桩的水平振动进行了求解, 得到了管桩桩顶的水平动力阻抗, 并分析了主要桩、土参数对管桩水平动力阻抗的影响。研究表明: 管桩内外半径、桩周土和桩芯土剪切模量比、泊松比之比对管桩水平动力阻抗的影响较大, 低频时液-固耦合系数比对管桩水平动力阻抗有一定的影响, 而阻尼比之比对管桩水平动力阻抗阻尼因子有一定的影响。

关 键 词: 饱和土; 管桩; 水平振动; 动力阻抗; 剪切模量

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Study of lateral dynamic impedance of pipe pile in saturated soil

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Abstract: In order to investigate the influence of main parameters of pile and soil on the lateral vibration of pipe pile in saturated soil, the soil is divided into saturated soil around pile and inner pile saturated soil, and coupling dynamic model of the saturated soil-pipe pile is established by using the control equations of theory of porous medium. By considering the boundaries of saturated soil around pile and inner pile saturated soil, the lateral vibrations of saturated soil around pile and inner pile core saturated soil are solved with the uncoupling method of potential function. The horizontal vibration of pipe pile in saturated soil is also solved by considering the forces of saturated soil around pile and inner pile saturated soil acting on pipe pile, and the lateral dynamic impedance of the pipe pile at pile head is obtained; the influence of main parameters on the lateral vibration of pipe pile in saturated soil is also analyzed. The research results indicate that the influence of the inner and outer radii of pipe pile, shear modulus ratio, Poisson' ratio of pile around saturated soil and pile core saturated soil on the lateral dynamic impedance of pipe pile in saturated soil is great; fluid-solid coefficient ratio has a certain influence on lateral dynamic impedance at low frequency; and the ratio of damping ratio has a certain influence on the damping factor of lateral dynamic impedance.

Key words: saturated soil; pipe pile; lateral vibration; dynamic impedance; shear modulus

1 引 言

桩基作为一种重要的基础形式, 在工程中得到了广泛的应用。由于桩基通常要承受各种动态荷载, 所以有关桩基振动特性的研究得到了足够的重视, 并取得了一定的研究成果^[1-4]。从 20 世纪中期开始, 管桩开始应用于铁道桥梁工程、建筑工程、高速公路、市政工程加固等工程领域。管桩按混凝土强度等级和壁厚分为预应力混凝土管桩、预应力混凝土

薄壁管桩和预应力高强混凝土管桩。在风荷载、地震荷载、动力机器荷载、高速列车荷载等荷载的作用下, 管桩将承受竖向、水平或扭转等动态荷载的作用, 因此, 在管桩的设计和施工中需要考虑管桩的动态特性, 需要了解相关参量对管桩振动特性的影响规律。目前, 针对管桩的振动特性的研究相对较少, 丁选明等^[5]针对轴对称均匀黏弹性地基中现浇薄壁管桩纵向振动的定解问题进行了求解。杨文领等^[6]考虑管桩与土体的相互作用, 研究了管桩的

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桩顶扭转刚度。石振明等^[7]系统地阐述了竖向/水平-回转耦合强迫振动和扭转自由振动等动力特性测试的原理和计算方法。需要引起注意的是, 由于土体是由固相、液相、气相三相组成的多孔介质, 如果把土体视为单相介质来考察管桩振动特性将与工程实际存在差距。当土骨架间的孔隙全部被水充满时, 则土体就是理想的两相饱和多孔介质。为了考虑桩基和土体渗透率的不同, 以及桩周饱和土和桩芯饱和土性质的不同对管桩振动的影响, 将桩周土和桩芯土体视为两相饱和土来研究管桩的振动特性则更加合理, 针对饱和土中管桩的纵向振动, 刘林超等^[8]在 Novak 平面应变的基础上对饱和土中管桩的纵向振动进行了初步研究。本文将在多孔介质理论和 Novak 平面应变假定的基础上, 分别求解桩周饱和土和桩芯饱和土的水平振动, 并在此基础上分析研究饱和土中管桩的水平动力阻抗。

2 模型与边界条件

考察图 1 所示饱和土中管桩的水平简谐振动, 管桩内径和外径分别为 r_1 、 r_2 , 桩的弹性模量、桩身体密度、横截面惯性矩分别为 E_p 、 ρ_p 、 I_p , 管桩将饱和土分成两部分, 即桩周饱和土和桩芯饱和土。桩周饱和土的液相和固相的体积分数分别为 n_0^F 和 n_0^S , 且 $n_0^F + n_0^S = 1$, 桩周饱和土骨架的剪切模量为 G_0 , 阻尼系数为 ξ_0 , 泊松比为 ν_0^S , 固相的表现密度为 ρ_0^S , 液相的表现密度为 ρ_0^F 。桩芯饱和土的液相和固相的体积分数分别为 n_1^F 和 n_1^S , 且 $n_1^F + n_1^S = 1$, 桩芯饱和土骨架的剪切模量为 G_1 , 阻尼系数为 ξ_1 , 泊松比为 ν_1^S , 固相的表现密度为 ρ_1^S , 液相的表现密度为 ρ_1^F 。设土体简谐振动的圆频率为 ω , 桩、土振动变形假定为小变形, 桩-土间无相对的滑移和脱离, 且桩土接触面不透水, 同时忽略土体的竖向位移。设无量纲化后的桩的水平位移幅值为 $\bar{u}_p$, 考虑桩-土接触面处的连续条件, 则有如下边界条件:

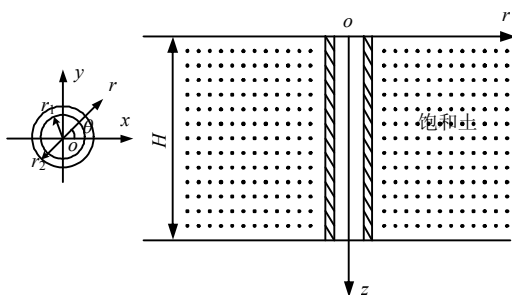


图 1 饱和土-管桩模型

Fig.1 Model of saturated soil-pipe pile

$$\left. \begin{aligned} \bar{u}_{Or}(\bar{r}_1, \theta) &= \bar{u}_p \cos \theta \\ \bar{u}_{O\theta}(\bar{r}_1, \theta) &= -\bar{u}_p \sin \theta \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} \bar{u}_{lr}(\bar{r}_2, \theta) &= \bar{u}_p \cos \theta \\ \bar{u}_{l\theta}(\bar{r}_2, \theta) &= -\bar{u}_p \sin \theta \end{aligned} \right\} \quad (2)$$

式中: $\bar{r}_1 = r_1 / H$, $\bar{r}_2 = r_2 / H$, $\bar{u}_{Or}$ 和 $\bar{u}_{O\theta}$ 为无量纲的桩周土径向和环向位移幅值; $\bar{u}_{lr}$ 和 $\bar{u}_{l\theta}$ 为无量纲的桩芯土径向和环向位移幅值; $\bar{u}_{Or} = u_{Or} / H$, $\bar{u}_{O\theta} = u_{O\theta} / H$, $\bar{u}_{lr} = u_{lr} / H$, $\bar{u}_{l\theta} = u_{l\theta} / H$; u_{Or} 和 $u_{O\theta}$ 为桩周土径向和环向位移幅值; u_{lr} 和 $u_{l\theta}$ 为桩芯土径向和环向位移幅值。对于无限远处, 土体位移为 0, 则有

$$\left. \begin{aligned} \bar{u}_{Or}(\bar{r}, \theta) &= 0, \bar{r} \rightarrow \infty \\ \bar{u}_{O\theta}(\bar{r}, \theta) &= 0, \bar{r} \rightarrow \infty \end{aligned} \right\} \quad (3)$$

设桩-土接触面不透水, 可知:

$$\frac{\partial \bar{P}_0}{\partial \bar{r}} = 0, \frac{\partial \bar{P}_1}{\partial \bar{r}} = 0 \quad (4)$$

3 桩周饱和土水平振动求解

对于桩周饱和土和桩芯饱和土采用基于连续介质混合物公理和体积分数概念的多孔介质理论来描述, 根据多孔介质理论, 则有桩周饱和土的三维控制方程为^[9-11]

$$\left. \begin{aligned} (\lambda_0^S + \mu_0^S) \text{grad div } \mathbf{u}_0^S + \mu_0^S \text{div grad } \mathbf{u}_0^S - \\ \text{grad } P_0 - \rho_0^S \ddot{\mathbf{u}}_0^S - \rho_0^F \ddot{\mathbf{u}}_0^F = 0 \\ -n_0^F \text{grad } P_0 - \rho_0^F \ddot{\mathbf{u}}_0^F - S_{0v}(\ddot{\mathbf{u}}_0^S - \ddot{\mathbf{u}}_0^F) = 0 \\ \text{div}(n_0^S \dot{\mathbf{u}}_0^S + n_0^F \dot{\mathbf{u}}_0^F) = 0 \end{aligned} \right\} \quad (5)$$

式中: $\mu_0^S = n_0^S G_0 (1 + 2i\xi_0)$; $\lambda_0^S = \frac{2\nu_0}{1-2\nu_0} \mu_0^S$ 为桩

周饱和土固相的表现复 Lamé 常数; $\mathbf{u}_0^S$ 和 $\mathbf{u}_0^F$ 为桩周饱和土固相和液相的位移矢量; P_0 为桩周饱和土孔隙流相压力; S_{0v} 为桩周饱和土的液-固耦合系数,

$S_{0v} = \frac{(n_0^F)^2 \gamma_0^{\text{LR}}}{k_0^L}$, 为表征桩周饱和土流-固两相相互

作用的液-固耦合系数; γ_0^{LR} 为桩周饱和土流相的重度; k_0^L 为桩周饱和土 Darcy 渗流系数, 可以通过孔隙流相压力和液-固耦合系数来反映水压力响应和渗透性的影响。由于管桩和饱和土组成的体系在简谐荷载作用下作稳态响应, 则各参量具有 $f = \tilde{f}e^{i\omega t}$ 的形式, $i = \sqrt{-1}$ 。忽略竖向位移, 将各参量代入式 (5), 消去方程两端的 $e^{i\omega t}$ 项, 并进行无量纲运算,

$\tilde{f}$ 的无量纲量记为 $\bar{f}$, 并令

$$\left. \begin{aligned} \bar{u}_{Or}^S &= \frac{\partial \Phi_0^S}{\partial \bar{r}} + \frac{1}{\bar{r}} \frac{\partial \Psi_0^S}{\partial \theta} \\ \bar{u}_{O\theta}^S &= \frac{1}{\bar{r}} \frac{\partial \Phi_0^S}{\partial \theta} - \frac{\partial \Psi_0^S}{\partial \bar{r}} \\ \bar{u}_{Or}^F &= \frac{\partial \Phi_0^F}{\partial \bar{r}} + \frac{1}{\bar{r}} \frac{\partial \Psi_0^F}{\partial \theta} \\ \bar{u}_{O\theta}^F &= \frac{1}{\bar{r}} \frac{\partial \Phi_0^F}{\partial \theta} - \frac{\partial \Psi_0^F}{\partial \bar{r}} \end{aligned} \right\} \quad (6)$$

式中: $\bar{r} = \frac{r}{H}$ 。利用式 (6) 可将式 (5) 用势函数解耦为

$$\left. \begin{aligned} \nabla^2 \Psi_0^S + \bar{\omega}^2 \Psi_0^S + \frac{\bar{\omega}^2}{\bar{\rho}} \Psi_0^F &= 0 \\ \Psi_0^S - \left(1 + \frac{i\bar{\omega}}{\bar{\rho}_0 s_{Ov}} \right) \Psi_0^F &= 0 \\ \nabla^2 \Phi_0^S + \frac{1-2\nu_0^S}{2(1-\nu_0^S)n_0^S(1+2i\xi_0)} \cdot \\ \left(-\bar{P}_0 + \bar{\omega}^2 \Phi_0^S + \frac{\bar{\omega}^2}{\bar{\rho}_0} \Phi_0^F \right) &= 0 \\ \Phi_0^S - \left(1 + \frac{\bar{\omega}i}{\bar{\rho}_0 s_{Ov}} \right) \Phi_0^F + \frac{n_0^F i}{\bar{\omega} s_{Ov}} \bar{P}_0 &= 0 \\ n_0^S \nabla^2 \Phi_0^S + n_0^F \nabla^2 \Phi_0^F &= 0 \end{aligned} \right\} \quad (7)$$

由式 (7) 的第 1、2 式可得

$$(\nabla^2 - q_0^2) \Psi_0^S = 0 \quad (8)$$

式中: $q_0^2 = -\left(1 + \frac{a_{01}}{\bar{\rho}_0} \right) \omega^2$, $a_{01} = \frac{\bar{\rho}_0 s_{Ov}}{\bar{\rho}_0 s_{Ov} + i\bar{\omega}}$, $\nabla^2 =$

$$\frac{\partial^2}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} + \frac{1}{\bar{r}^2} \frac{\partial^2}{\partial \theta^2}, \quad \bar{P}_0 = \frac{P_0}{v_{Os} \rho_0^S}, \quad s_{Ov} = \frac{H s_{Ov}}{v_{Os} \rho_0^S},$$

$$\bar{\omega} = \frac{H\omega}{v_{Os}}, \quad \bar{\rho}_0 = \frac{\rho_0^S}{\rho_0^F}, \quad v_{Os} = \sqrt{\frac{G_0}{\rho_0^S}} \text{ 为桩周饱和土固}$$

相骨架的剪切波速。采用分离变量法求解式 (8), 并考虑边界条件 (1) 可得

$$\Psi_0^S = A_1 K_1(q_0 \bar{r}) \sin \theta \quad (9)$$

式中: $K_1(\cdot)$ 为 1 阶第 2 类虚宗量 Bessel 函数; A_1 为待定常数。考虑式 (7) 的第 2 式和式 (9), 有

$$\Psi_0^F = a_{01} A_1 K_1(q_0 \bar{r}) \sin \theta \quad (10)$$

由式 (7) 第 3、4 式可得

$$\nabla^2 \Phi_0^S + a_{02} \Phi_0^S + a_{03} \Phi_0^F = 0 \quad (11)$$

其中:

$$a_{02} = \frac{1-2\nu_0^S}{2(1-\nu_0^S)n_0^S(1+2i\xi_0)} \left(\bar{\omega}^2 - \frac{i\bar{\omega}s_{Ov}}{n_0^F} \right) \quad (12)$$

$$a_{03} = \frac{1-2\nu_0^S}{2(1-\nu_0^S)n_0^S(1+2i\xi_0)} \left[\frac{\bar{\omega}^2}{\bar{\rho}_0} + \frac{i\bar{\omega}s_{Ov}}{n_0^F} \left(1 + \frac{i\bar{\omega}}{\bar{\rho}_0 s_{Ov}} \right) \right] \quad (13)$$

对式 (11) 两端进行 ∇^2 运算, 并结合式 (7) 的第 5 式, 由分离变量法得

$$\nabla^2 \Phi_0^S = A_2 K_1(s_0 \bar{r}) \cos \theta \quad (14)$$

式中: $s_0^2 = -\left(a_{02} - \frac{n_0^S}{n_0^F} a_{03} \right)$; A_2 为待定系数。设 $\Phi_0^S =$

$F(\bar{r}) \cos \theta$, 代入式 (12), 有

$$\frac{d^2 F}{d\bar{r}^2} + \frac{1}{\bar{r}} \frac{dF}{d\bar{r}} - \frac{1}{\bar{r}^2} F = A_2 K_1(s_0 \bar{r}) \quad (15)$$

求解式 (15), 有

$$F(\bar{r}) = A_3 \bar{r} + \frac{A_4}{\bar{r}} + \frac{K_1(s_0 \bar{r})}{s_0^2} A_2 \quad (16)$$

式中: A_3 、 A_4 均为待定常数。考虑边界条件 (3), 则有

$$\Phi_0^S = \left[\frac{A_4}{\bar{r}} + \frac{K_1(s_0 \bar{r})}{s_0^2} A_2 \right] \cos \theta \quad (17)$$

由式 (7)、(11)、(14) 和式 (17), 可得

$$\begin{aligned} \Phi_0^F &= -\frac{1}{a_{03}} A_2 K_1(s_0 \bar{r}) \cos \theta - \\ &\frac{a_{02}}{a_{03}} \left[\frac{A_4}{\bar{r}} + \frac{K_1(s_0 \bar{r})}{s_0^2} A_2 \right] \cos \theta \end{aligned} \quad (18)$$

$$\begin{aligned} P_0 &= \frac{i\bar{\omega}s_{Ov}}{n_0^F} \left\{ \frac{1}{a_{01}a_{03}} A_2 K_1(s_0 \bar{r}) + \right. \\ &\left. \frac{a_{02} + a_{01}a_{03}}{a_{01}a_{03}} \left[\frac{A_4}{\bar{r}} + \frac{K_1(s_0 \bar{r})}{s_0^2} A_2 \right] \right\} \cos \theta \end{aligned} \quad (19)$$

进而可得径向和环向位移为

$$\bar{u}_{Or}^S = \left[-\frac{A_4}{\bar{r}^2} - \frac{K_1(s_0 \bar{r})}{s_0^2 \bar{r}} A_2 - \frac{K_0(s_0 \bar{r})}{s_0} A_2 + \frac{K_1(q_0 \bar{r})}{\bar{r}} A_1 \right] \cos \theta \quad (20)$$

$$\begin{aligned} \bar{u}_{O\theta}^S &= \left[-\frac{A_4}{\bar{r}^2} - \frac{K_1(s_0 \bar{r})}{s_0^2 \bar{r}} A_2 + \frac{K_1(q_0 \bar{r})}{\bar{r}} A_1 + \right. \\ &\left. q_0 K_0(q_0 \bar{r}) A_1 \right] \sin \theta \end{aligned} \quad (21)$$

当 $\theta = 0$, $\theta = \frac{\pi}{2}$ 时, 由边界条件 (1)、(3) 有

$$s_0^2(a_{02} + a_{01}a_{03})A_4 + (s_0^2 + a_{02} + a_{01}a_{03})\bar{r}_2 \cdot [K_1(s_0\bar{r}_2) + s_0\bar{r}_2K_0(s_0\bar{r}_2)]A_2 = 0 \quad (22)$$

$$-\frac{A_4}{\bar{r}_2^2} - \frac{K_1(s_0\bar{r}_2)}{s_0^2\bar{r}_2}A_2 - \frac{K_0(s_0\bar{r}_2)}{s_0}A_2 + \frac{K_1(q_0\bar{r}_2)}{\bar{r}_2}A_1 = \bar{u}_p \quad (23)$$

$$-\frac{A_4}{\bar{r}_2^2} - \frac{K_1(s_0\bar{r}_2)}{s_0^2\bar{r}_2}A_2 + \frac{K_1(q_0\bar{r}_2)}{\bar{r}_2}A_1 + q_0K_0(q_0\bar{r}_2)A_1 = -\bar{u}_p \quad (24)$$

由方程 (22) ~ (24) 确定待定系数 $A_2 = \frac{D_2}{D_1}\bar{u}_p$,

$$A_4 = \frac{D_2D_3}{D_1}\bar{u}_p, \quad A_1 = -\frac{2s_0D_1 + K_0(s_0\bar{r}_2)D_2}{q_0s_0K_0(q_0\bar{r}_2)D_1}\bar{u}_p. \text{ 其中:}$$

$$D_2 = (a_{02} + a_{01}a_{03})s_0[2K_1(q_0\bar{r}_2) + q_0\bar{r}_2K_0(q_0\bar{r}_2)] \quad (25)$$

$$D_3 = -\frac{(s_0^2 + a_{02} + a_{01}a_{03})\bar{r}_2[K_1(s_0\bar{r}_2) + s_0\bar{r}_2K_0(s_0\bar{r}_2)]}{s_0^2(a_{02} + a_{01}a_{03})} \quad (26)$$

$$D_1 = s_0q_0[K_1(s_0\bar{r}_2) + s_0\bar{r}_2K_0(s_0\bar{r}_2)]K_0(q_0\bar{r}_2) - (a_{02} + a_{01}a_{03})K_1(q_0\bar{r}_2)K_0(s_0\bar{r}_2) \quad (27)$$

4 桩芯饱和土水平振动求解

同样地, 由多孔介质理论可知, 桩芯饱和土的三维控制方程为

$$\left. \begin{aligned} &(\lambda_1^S + \mu_1^S)\text{grad div } \mathbf{u}_1^S + \mu_1^S \text{div grad } \mathbf{u}_1^S \\ &-\text{grad } \mathbf{P}_1 - \rho_1^S \ddot{\mathbf{u}}_1^S - \rho_1^F \ddot{\mathbf{u}}_1^F = 0 \\ &-n_1^F \text{grad } \mathbf{P}_1 - \rho_1^F \ddot{\mathbf{u}}_1^F - S_{lv}(\ddot{\mathbf{u}}_1^S - \ddot{\mathbf{u}}_1^F) = 0 \\ &\text{div}(n_1^S \dot{\mathbf{u}}_1^S + n_1^F \dot{\mathbf{u}}_1^F) = 0 \end{aligned} \right\} \quad (28)$$

式中: $\mu_1^S = n_1^S G_1(1 + 2i\xi_1)$; $\lambda_1^S = \frac{2v_1^S}{1 - 2v_1^S}\mu_1^S$, 为桩芯

饱和土固相的表观复 Lamé 常数; $\mathbf{u}_1^S$ 和 $\mathbf{u}_1^F$ 为桩芯饱和土固相和液相的位移矢量; P_1 为桩芯饱和土的孔隙流相压力; S_{lv} 为桩芯饱和土的液-固耦合系数,

$S_{lv} = \frac{(n_1^F)^2 \gamma_1^{\text{LR}}}{k_1^L}$; γ_1^{LR} 为桩芯饱和土流相的重度; k_1^L 为

桩芯饱和土的 Darcy 渗流系数。

同样地, 忽略竖向位移, 将各参量 $f = \tilde{f}e^{i\omega t}$ 代入式 (28), 消去方程两端的 $e^{i\omega t}$ 项, 进行无量纲运

算, 并令

$$\left. \begin{aligned} \bar{u}_{lv}^S &= \frac{\partial \Phi_1^S}{\partial \bar{r}} + \frac{1}{\bar{r}} \frac{\partial \Psi_1^S}{\partial \theta} \\ \bar{u}_{lv}^F &= \frac{\partial \Phi_1^F}{\partial \bar{r}} + \frac{1}{\bar{r}} \frac{\partial \Psi_1^F}{\partial \theta} \\ \bar{u}_{1\theta}^S &= \frac{1}{\bar{r}} \frac{\partial \Phi_1^S}{\partial \theta} - \frac{\partial \Psi_1^S}{\partial \bar{r}} \\ \bar{u}_{1\theta}^F &= \frac{1}{\bar{r}} \frac{\partial \Phi_1^F}{\partial \theta} - \frac{\partial \Psi_1^F}{\partial \bar{r}} \end{aligned} \right\} \quad (29)$$

利用式 (29) 对式 (28) 解耦得

$$\left. \begin{aligned} &\nabla^2 \Psi_1^S + \bar{\omega}^2 \Psi_1^S + \frac{\bar{\omega}^2}{\rho_1} \Psi_1^F = 0 \\ &\Psi_1^S - \left(1 + \frac{i\bar{\omega}}{\bar{\rho}_1 s_{lv}}\right) \Psi_1^F = 0 \\ &\nabla^2 \Phi_1^S + \frac{1 - 2v_1^S}{2(1 - v_1^S)n_1^S(1 + 2i\xi_1)} \cdot \\ &\quad \left(-\bar{P}_1 + \bar{\omega}^2 \Phi_1^S + \frac{\bar{\omega}^2}{\bar{\rho}_1} \Phi_1^F\right) = 0 \\ &\Phi_1^S - \left(1 + \frac{i\bar{\omega}}{\bar{\rho}_1 s_{lv}}\right) \Phi_1^F + \frac{n_1^F i}{\bar{\omega} s_{lv}} \bar{P}_1 = 0 \\ &n_1^S \nabla^2 \Phi_1^S + n_1^F \nabla^2 \Phi_1^F = 0 \end{aligned} \right\} \quad (30)$$

$$\text{令 } \bar{\omega} = \frac{H\omega}{v_{0s}}, \quad \bar{P}_1 = \frac{P_1}{v_{ls}^2 \rho_1^S}, \quad s_{lv} = \frac{HS_{lv}}{v_{ls} \rho_1^S}, \quad \bar{\rho}_1 =$$

$$\frac{\rho_1^S}{\rho_1^F}, \quad v_{ls} = \sqrt{\frac{G_1}{\rho_1^S}}, \quad v_{0s} = \sqrt{\frac{G_1 \rho_0^S}{G_0 \rho_1^S}} = \sqrt{\frac{G}{\rho^S}}, \quad v = \frac{v_1^S}{v_0^S},$$

$$\xi = \frac{\xi_1^S}{\xi_0^S}, \quad G = \frac{G_1^S}{G_0^S}, \quad \rho^S = \frac{\rho_1^S}{\rho_0^S}, \quad \rho^F = \frac{\rho_1^F}{\rho_0^F}, \quad n^S = \frac{n_1^S}{n_0^S},$$

$$\frac{s_{lv}}{s_{0v}} = S_v, \quad \frac{\bar{\rho}_1}{\rho_0} = \frac{\rho^S}{\rho^F}, \text{ 则式 (30) 改写为}$$

$$\left. \begin{aligned} &\nabla^2 \Psi_1^S + \frac{\rho^S}{G} \bar{\omega}^2 \Psi_1^S + \frac{\rho^F \bar{\omega}^2}{G \bar{\rho}_0} \Psi_1^F = 0 \\ &\Psi_1^S - \left(1 + \frac{i\bar{\omega} \rho^F}{\bar{\rho}_0 S_v s_{0v}}\right) \Psi_1^F = 0 \\ &\nabla^2 \Phi_1^S + \frac{1 - 2vv_0^S}{2(1 - vv_0^S)n^S n_0^S(1 + 2i\xi\xi_0)} \cdot \\ &\quad \left(-\bar{P}_1 + \frac{\rho^S}{G} \bar{\omega}^2 \Phi_1^S + \frac{\bar{\omega}^2 \rho^F}{G \bar{\rho}_0} \Phi_1^F\right) = 0 \\ &\Phi_1^S - \left(1 + \frac{i\bar{\omega} \rho^F}{\bar{\rho}_0 S_v s_{0v}}\right) \Phi_1^F + \frac{G n^F n_0^F i}{S_v s_{0v} \bar{\omega}} \bar{P}_1 = 0 \\ &n^S n_0^S \nabla^2 \Phi_1^S + n^F n_0^F \nabla^2 \Phi_1^F = 0 \end{aligned} \right\} \quad (31)$$

由式 (31) 的第 1、2 式可得

$$(\nabla^2 - q_1^2)\Psi_1^S = 0 \quad (32)$$

这里, $q_1^2 = -\left(\frac{\rho^S}{G} + \frac{\rho^F a_{11}}{G\rho_0}\right)\omega^2$, $a_{11} = \frac{\bar{\rho}_0 S_v s_{0v}}{\bar{\rho}_0 S_v s_{0v} + i\rho^F \bar{\omega}}$,

$\nabla^2 = \frac{\partial^2}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} + \frac{1}{\bar{r}^2} \frac{\partial^2}{\partial \theta^2}$ 。同样, 采用分离变量法求解式 (32), 并考虑边界条件 (2) 以及 $\bar{r} = 0$ 时位移的有界性可得

$$\Psi_1^S = F_1 I_1(q\bar{r}) \sin \theta \quad (33)$$

式中: $I_1(\bullet)$ 为 1 阶第 1 类虚宗量 Bessel 函数; F_1 为待定常数。考虑由式 (31) 的第 2 式和式 (33) 可得

$$\Psi_1^F = a_{11} F_1 I_1(q\bar{r}) \sin \theta \quad (34)$$

由式 (31) 第 3、4 式可得

$$\nabla^2 \Phi_1^S + a_{12} \Phi_1^S + a_{13} \Phi_1^F = 0 \quad (35)$$

其中:

$$a_{12} = \frac{1 - 2\nu v_0^S}{2(1 - \nu v_0^S)n^S n_0^S(1 + 2i\xi\xi_0)} \left(\frac{\rho^S}{G} \bar{\omega}^2 - \frac{iS_v s_{0v} \bar{\omega}}{Gn^F n_0^F} \right) \quad (36)$$

$$a_{13} = \frac{1 - 2\nu v_0^S}{2(1 - \nu v_0^S)n^S n_0^S(1 + 2i\xi\xi_0)} \left[\frac{\bar{\omega}^2 \rho^F}{G\rho_0} + \frac{iS_v s_{0v} \bar{\omega}}{Gn^F n_0^F} \left(1 + \frac{i\bar{\omega} \rho^F}{\bar{\rho}_0 S_v s_{0v}} \right) \right] \quad (37)$$

对式 (35) 两端进行 ∇^2 运算, 并考虑式 (31) 的第 5 式, 利用分离变量法得

$$\nabla^2 \Phi_1^S = F_2 I_1(s_1 \bar{r}) \cos \theta \quad (38)$$

式中: $s_1^2 = -\left(a_{12} - \frac{n_1^S}{n_1^F} a_{13}\right)$; F_2 为待定系数。令 $\Phi_1^S = Y(\bar{r}) \cos \theta$, 代入式 (38) 可得

$$\frac{d^2 Y}{d\bar{r}^2} + \frac{1}{\bar{r}} \frac{dY}{d\bar{r}} - \frac{1}{\bar{r}^2} Y = B_2 I_1(s\bar{r}) \quad (39)$$

求解方程 (39) 得

$$Y(r) = F_3 \bar{r} + \frac{F_4}{\bar{r}} + \frac{I_1(s_1 \bar{r})}{s_1^2} F_2 \quad (40)$$

式中: F_3 、 F_4 均为待定常数。考虑 $\bar{r} = 0$ 时位移的有界性, 可得

$$\Phi_1^S = \left[F_3 \bar{r} + \frac{I_1(s_1 \bar{r})}{s_1^2} F_2 \right] \cos \theta \quad (41)$$

由式 (31)、(35)、(38) 和式 (41) 可得

$$\Phi_1^F = -\frac{1}{a_{13}} F_2 I_1(s_1 \bar{r}) \cos \theta - \frac{a_{12}}{a_{13}} \left[F_3 \bar{r} + \frac{I_1(s_1 \bar{r})}{s_1^2} F_2 \right] \cos \theta \quad (42)$$

$$P_1 = \frac{i\bar{\omega} S_v s_{0v}}{Gn^F n_0^F} \left\{ \frac{1}{a_{11} a_{13}} F_2 I_1(s_1 \bar{r}) + \frac{a_{12} + a_{11} a_{13}}{a_{11} a_{13}} \left[F_3 \bar{r} + \frac{I_1(s_1 \bar{r})}{s_1^2} F_2 \right] \right\} \cos \theta \quad (43)$$

由此, 可得桩芯饱和土的径向和环向位移为

$$\bar{u}_r^S = \left[F_3 - \frac{I_1(s_1 \bar{r})}{s_1^2 \bar{r}} F_2 + \frac{I_0(s_1 \bar{r})}{s_1} F_2 + \frac{I_1(q_1 \bar{r})}{\bar{r}} F_1 \right] \cos \theta \quad (44)$$

$$\bar{u}_{\theta}^S = \left[-F_3 - \frac{I_1(s_1 \bar{r})}{s_1^2 \bar{r}} F_2 + \frac{I_1(q_1 \bar{r})}{\bar{r}} F_1 - q_1 I_0(q_1 \bar{r}) F_1 \right] \sin \theta \quad (45)$$

由边界条件 (2)、(3) 可得

$$s_1^2 (a_{12} + a_{11} a_{13}) \bar{r}_1 F_3 + (s_1^2 + a_{12} + a_{11} a_{13}) \cdot [s_1 \bar{r}_1 I_0(s_1 \bar{r}_1) - I_1(s_1 \bar{r}_1)] F_2 = 0 \quad (46)$$

$$F_3 - \frac{I_1(s_1 \bar{r}_1)}{s_1^2 \bar{r}_1} F_2 + \frac{I_0(s_1 \bar{r}_1)}{s_1} F_2 + \frac{I_1(q_1 \bar{r}_1)}{\bar{r}_1} F_1 = \bar{u}_p \quad (47)$$

$$-F_3 - \frac{I_1(s_1 \bar{r}_1)}{s_1^2 \bar{r}_1} F_2 + \frac{I_1(q_1 \bar{r}_1)}{\bar{r}_1} F_1 - q_1 I_0(q_1 \bar{r}_1) F_1 = -\bar{u}_p \quad (48)$$

由式 (46) ~ (48) 确定待定系数:

$$\left. \begin{aligned} F_2 &= \frac{Y_2}{Y_1} \bar{u}_p \\ F_3 &= \frac{Y_2 Y_3}{Y_1} \bar{u}_p \\ F_1 &= \frac{s_1 \bar{r}_1 I_0(s_1 \bar{r}_1) - 2I_1(s_1 \bar{r}_1)}{q_1 s_1^2 \bar{r}_1 I_0(q_1 \bar{r}_1) - 2s_1^2 I_1(q_1 \bar{r}_1)} \frac{Y_2}{Y_1} \bar{u}_p \end{aligned} \right\} \quad (49)$$

$$\left. \begin{aligned} Y_2 &= s_1^2 \bar{r}_1 (a_{12} + a_{11} a_{13}) [q_1 \bar{r}_1 I_0(q_1 \bar{r}_1) - 2I_1(q_1 \bar{r}_1)] \\ Y_3 &= -\frac{(s_1^2 + a_{12} + a_{11} a_{13}) [s_1 \bar{r}_1 I_0(s_1 \bar{r}_1) - I_1(s_1 \bar{r}_1)]}{s_1^2 (a_{12} + a_{11} a_{13}) \bar{r}_1} \\ Y_1 &= (a_{12} + a_{11} a_{13}) I_1(q_1 \bar{r}_1) [s_1 \bar{r}_1 I_0(s_1 \bar{r}_1) - 2I_1(s_1 \bar{r}_1)] - s_1^2 [s_1 \bar{r}_1 I_0(s_1 \bar{r}_1) - I_1(s_1 \bar{r}_1)] [q_1 \bar{r}_1 I_0(q_1 \bar{r}_1) - 2I_1(q_1 \bar{r}_1)] \end{aligned} \right\} \quad (50)$$

5 饱和土中管桩的水平振动求解

根据桩周饱和土和桩芯饱和土土体的径向、剪

切应力, 可以求得桩周饱和土对管桩的无量纲水平动反力幅值为

$$F_0 = \int_0^{2\pi} \left[(P_0 - \bar{\sigma}_{Or}^S) \cos \theta + \bar{\tau}_{Or\theta}^S \sin \theta \right] \Big|_{\bar{r}=\bar{r}_2} \bar{r}_2 d\theta = \pi f_0 \bar{u}_p \quad (51)$$

其中:

$$f_0 = \bar{r}_2 n_0^S (1 + 2i\xi_0) \left[\frac{2\nu_0^S - 2}{1 - 2\nu_0^S} K_1(s_0 \bar{r}_2) A_2 - q_0^2 K_1(q_0 \bar{r}_2) A_1 + P_0(\bar{r}_2) \right] / \bar{u}_p \quad (52)$$

式中: $\bar{\sigma}_{Or}^S$ 和 $\bar{\tau}_{Or\theta}^S$ 分别为桩周饱和土固相土骨架无量纲径向和剪切应力幅值。桩芯饱和土对管桩的无量纲水平动反力幅值为

$$F_1 = \int_0^{2\pi} \left[(P_1 - \bar{\sigma}_{lr}^S) \cos \theta + \bar{\tau}_{lr\theta}^S \sin \theta \right] \Big|_{\bar{r}=\bar{r}_0} \bar{r}_0 d\theta = \pi f_1 \bar{u}_p \quad (53)$$

其中:

$$f_1 = \bar{r}_1 n_0^S G (1 + 2i\xi_0) \left[\frac{2\nu_0^S - 2}{1 - 2\nu_0^S} I_1(s_1 \bar{r}_1) F_2 - q_1^2 K_1(q_1 \bar{r}_1) F_1 + P_1(\bar{r}_1) \right] / \bar{u}_p \quad (54)$$

式中: $\bar{\sigma}_{lr}^S$ 和 $\bar{\tau}_{lr\theta}^S$ 为桩芯饱和土固相土骨架无量纲径向和剪切应力幅值。

考虑桩周饱和土和桩芯饱和土对桩的作用, 可以建立无量纲化的饱和土中管桩的水平振动微分方程为

$$\frac{d^4 \bar{u}_p(\bar{z})}{d\bar{z}^4} + 4\lambda^4 \bar{u}_p(\bar{z}) = 0 \quad (55)$$

式中: $\bar{u}_p(\bar{z})$ 为无量纲化后的管桩水平位移的幅值;

$$\lambda^4 = \frac{f_0 + f_1 - \bar{\rho}_p (\bar{r}_2^2 - \bar{r}_1^2) \bar{\omega}^2}{\bar{E}_p (\bar{r}_2^4 - \bar{r}_1^4)}, \quad \bar{\rho}_p = \frac{\rho_p}{\rho_0^S}, \quad \bar{E}_p = \frac{E_p}{G_0}.$$

利用初参数法可以将式 (55) 的解表示为^[12]

$$u_p(\bar{z}) = -U_0 F_1(\lambda \bar{z}) + \frac{\phi_0}{\lambda} F_2(\lambda \bar{z}) - \frac{M_0}{\frac{\pi}{4} \bar{E}_p \lambda^2 (\bar{r}_2^4 - \bar{r}_1^4)} \cdot F_3(\lambda \bar{z}) - \frac{Q_0}{\frac{\pi}{4} \bar{E}_p \lambda^3 (\bar{r}_2^4 - \bar{r}_1^4)} F_4(\lambda \bar{z}) \quad (56)$$

式中: U_0 、 ϕ_0 、 Q_0 、 M_0 均为管桩桩顶端的水平位移、转角、剪力和弯矩; F_1 、 F_2 、 F_3 、 F_4 均为克雷洛夫函数。同时假定管桩底固定, 亦即管桩为端

承摩擦桩, 则有如下边界条件:

$$\bar{u}_p(\bar{z}) \Big|_{\bar{z}=1} = 0, \quad \frac{d\bar{u}_p(\bar{z})}{d\bar{z}} \Big|_{\bar{z}=1} = 0 \quad (57)$$

由式 (56)、(57) 得

$$Q_0 = \frac{\pi}{4} \bar{E}_p (\bar{r}_2^4 - \bar{r}_1^4) \left\{ \frac{\lambda^3 (F_1 F_2 + 4F_3 F_4)}{F_3^2 - F_2 F_4} U_0 - \frac{\lambda^2 (F_2^2 - F_1 F_3)}{F_3^2 - F_2 F_4} \phi_0 \right\} \quad (58)$$

当管桩桩顶转角固定, 亦即转角为 0 时, 产生单位水平位移时所需的剪力, 即为管桩的水平动力阻抗 K_{hh} , 即

$$K_{hh} = \frac{Q_0}{U_0} = \frac{\pi}{4} \bar{E}_p (r_2^4 - r_1^4) \cdot \frac{\lambda^3 [F_1(\lambda) F_2(\lambda) + 4F_3(\lambda) F_4(\lambda)]}{F_3^2(\lambda) - F_2(\lambda) F_4(\lambda)} = f_{h1} + i\bar{\omega} f_{h2} \quad (59)$$

式中: f_{h1} 为管桩的水平动力阻抗刚度因子; f_{h2} 为管桩的水平动力阻抗阻尼因子。

6 桩、土参数对管桩水平动力阻抗影响

桩顶的水平动力阻抗能够反映出桩的振动特性, 由于管桩桩周土和桩芯土体对管桩的作用, 管桩内、外半径和土体参数将对管桩水平振动特性有影响, 这里通过参数分析来讨论桩、土参数对管桩水平动力阻抗的影响, 重点分析管桩内、外半径, 桩芯土和桩周土剪切模量比、液-固耦合系数比、泊松比、阻尼系数比等对饱和土中管桩水平动力阻抗的影响, 相关参数的取值为: $\bar{r}_1 = 1/25$, $\bar{r}_2 = 1/20$, $\bar{\rho}_0 = 2$, $s_{ov} = 0.01$, $\nu_{0s} = 0.3$, $n_0^S = 0.65$, $n_0^F = 0.35$, $\xi_0 = 0.05$, $\bar{E}_p = 1000$, $\bar{\rho}_p = 2.5$, $S_v = 1$, $\rho^F = 1$, $\rho^S = 1$, $G = 1$, $\nu = 1$, $n^F = 1$, $n^S = 1$, $\xi = 1$ 。管桩内、外径对饱和土中管桩水平动力阻抗的影响见图 2、3, 可以看出, 在管桩内径一定的情况下, 管桩的外径越大, 亦即管桩越厚, 管桩的水平动力阻抗刚度因子和阻尼因子都越大, 这里因为管桩外半径越大, 桩周土提供的作用力越大, 从而导致管桩水平动力阻抗越大; 在外径一定, 减小内径时, 在一定的范围内, 减小内径即增加管桩厚度可以提高管桩的水平阻抗刚度因子和阻尼因子, 但当内径很小时, 由于接近实心桩, 此时减小内径对桩的振动影响不大。桩芯饱和土固相土骨架剪切模量比和泊松比之比对管桩水平动力阻抗的影响如图 4、5

所示, 桩芯饱和土的剪切模量和泊松比越大, 管桩水平动力阻抗刚度因子和阻尼因子都越大, 可见桩芯饱和土的存在对饱和土管桩的影响还是比较大的。而桩芯的饱和土和桩周饱和土的液-固耦合系数对管桩水平动力阻抗的影响相对较小, 见图 6, 且在频率相对较低时液固耦合系数的影响对管桩水平

动力阻抗的影响较频率较高时要大, 饱和土液-固耦合系数越大, 水平动力阻抗刚度因子越小, 阻尼因子则越大。而饱和土阻尼比之比对管桩水平动力阻抗阻尼因子的影响较大 (见图 7), 而对水平动力阻抗刚度因子几乎没有影响, 且桩芯饱和土的阻尼比之比越大, 管桩水平动力阻抗的阻尼因子越大。

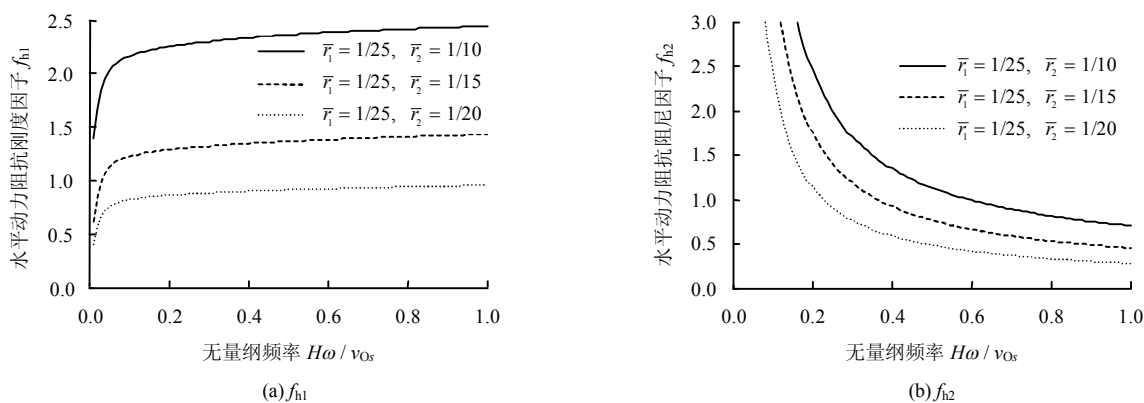


图 2 管桩外径对管桩水平动力阻抗的影响

Fig.2 Influence of outside diameters on lateral dynamic impedance of pipe pile

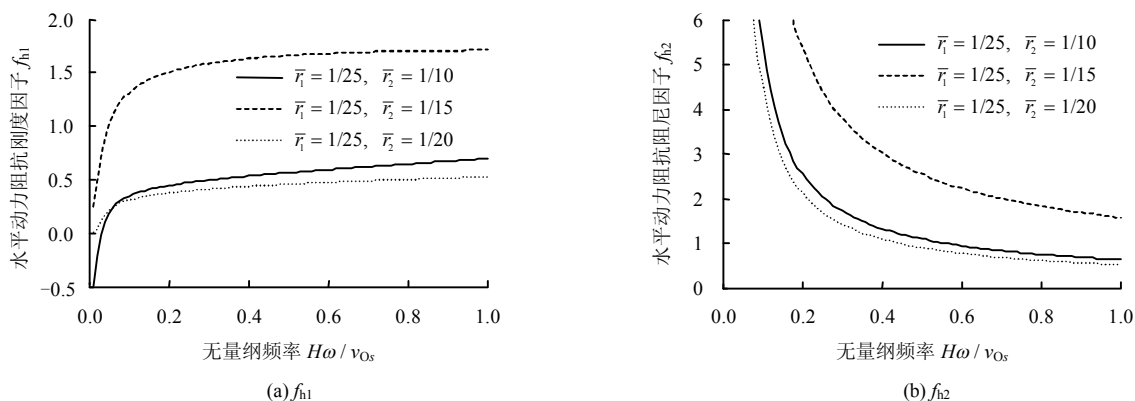


图 3 管桩内径对管桩水平动力阻抗的影响

Fig.3 Influence of inside diameters on lateral dynamic impedance of pipe pile

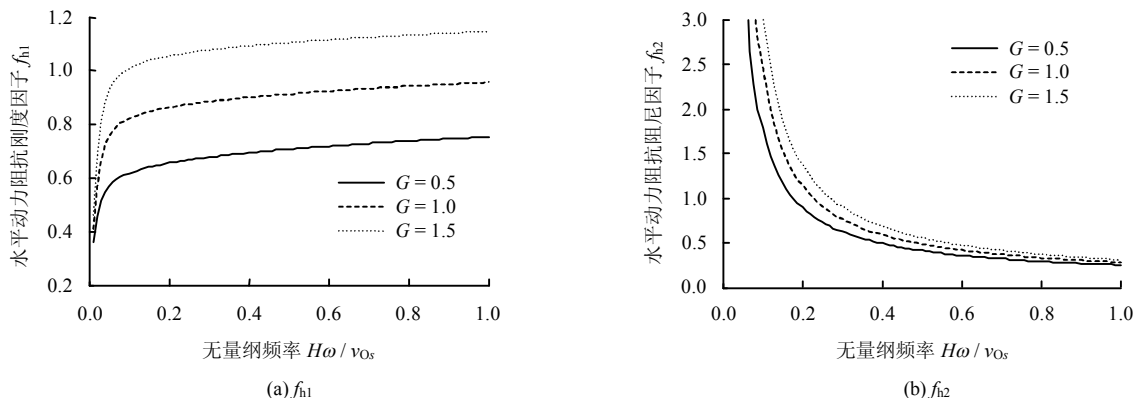


图 4 剪切模量比对管桩水平动力阻抗的影响

Fig.4 Influence of shear modulus ratio on lateral dynamic impedance of pipe pile

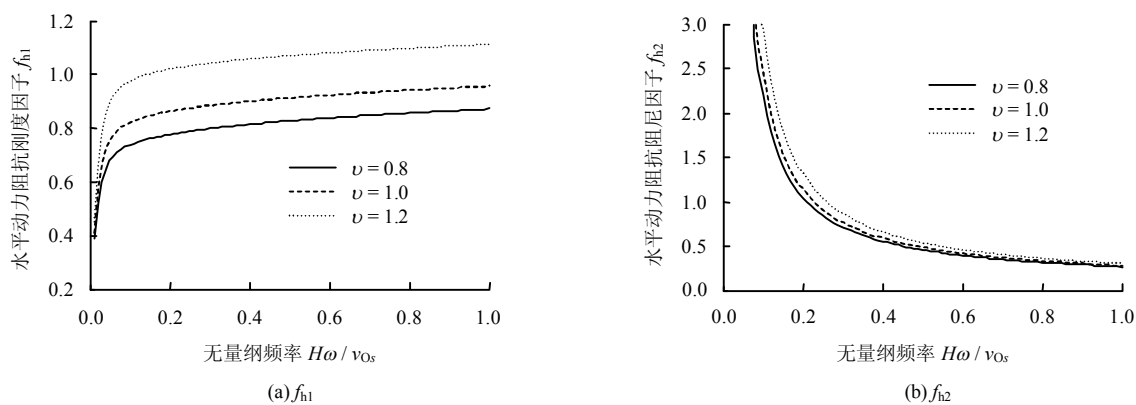


图5 泊松比对管桩水平动力阻抗的影响

Fig.5 Influence of Poisson' ratio on lateral dynamic impedance of pipe pile

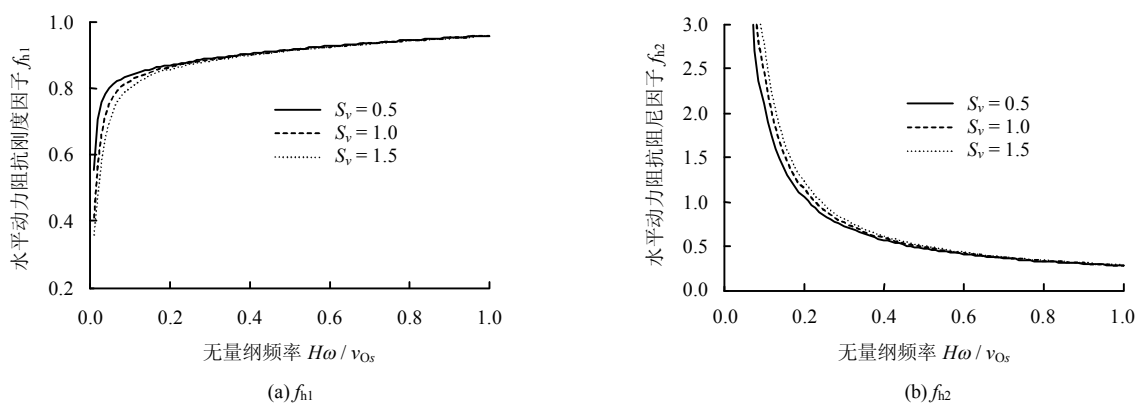


图6 液固耦合系数比对管桩水平动力阻抗的影响

Fig.6 Influence of liquid-solid coupling coefficient ratio on lateral dynamic impedance of pipe pile

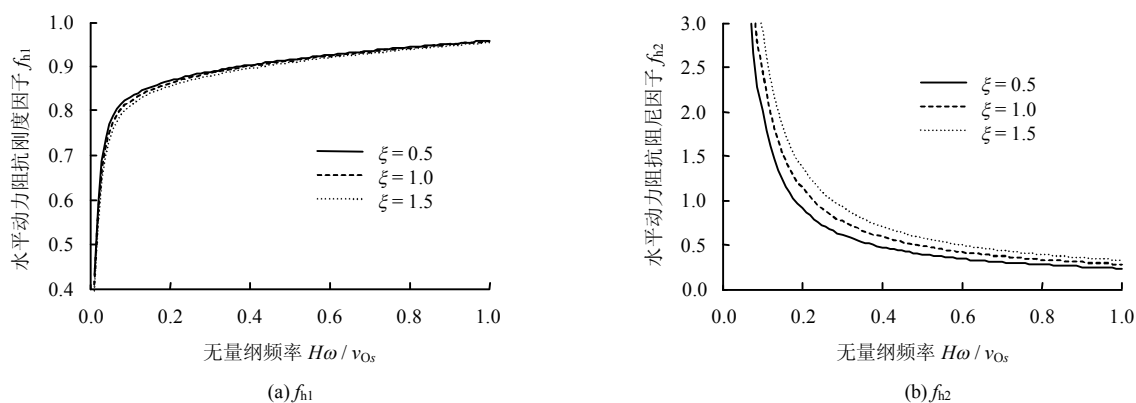


图7 阻尼比之比对管桩水平动力阻抗的影响

Fig.7 Influence of ratio of damping ratio on lateral dynamic impedance of pipe pile

7 结 论

为了更为符合工程实际, 将桩周和桩芯土体视为两相饱和和多孔介质, 借助势函数对桩周和桩芯饱和土的控制方程进行解耦, 在考虑桩周和桩芯土体边界条件的基础上求解了桩周和桩芯饱和土的水平振动, 在考虑桩-土相互作用的基础上研究了饱和土

中管桩的水平振动, 在对桩顶水平动力阻抗分析的基础上, 主要得到以下结论:

(1) 在同内径的情况下, 管桩壁越厚, 也就是厚壁的管桩, 其动力阻抗较大, 土体对管桩的约束作用越大。

(2) 桩芯饱和土的存在对饱和土管桩水平振动有较大的影响, 在设计和施工中不能忽略桩芯饱和

土的作用。

(3) 饱和土固相土骨架的剪切模量和泊松比对饱和土中管桩的水平动力阻抗影响较大, 而液固耦合系数和阻尼比的影响相对较小, 且液耦合系数对管桩水平振动的影响还与频率有一定的关系。

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